

## **Autonomy Underwater: Ocean Sampling by Autonomous Underwater Vehicles**

Derek Paley

University of Maryland

The ocean is large and opaque to electromagnetic waves used for communication and navigation, making autonomy underwater essential because of intermittent, low-bandwidth data transmission and inaccurate underwater positioning. Moreover, the vastness of the ocean coastlines and basins renders traditional sampling time-consuming and expensive; one typically must make do with sparse observations, which leads to the question of where to take those measurements. Further complicating the problem, circulating ocean processes pertinent to national defense and climate change span a range of small-to-large space and time scales, necessitating multiple sampling platforms or vehicles, ideally possessing long endurance. Many effective vehicle designs are not propeller-drive, but rather buoyancy driven; becoming heavy or light relative to the surrounding seawater generates vertical motion suitable for collecting profiles of hydrographic properties like salinity, density, and temperature. Buoyancy-driven vehicles drift with the currents unless they have wings, in which case their vertical motion is converted to horizontal motion via lift, like an air glider. The capacity to maneuver relative to the flow gives rise to challenges in cooperative control and adaptive sampling with the following recursive property: vehicles collect measurements of the ocean currents in order to estimate it; the estimate is used to guide the collection of subsequent measurements along sampling trajectories subject to currents that may be as large as the platform's through-water speed.

Approaches to adaptive sampling of continuous environmental processes are distinguished by the characterization of the estimated process as statistical or dynamical. A

statistical characterization comprises spatial and temporal decorrelation scales, which for a nonstationary process may themselves vary in space and time. Presuming these scales are known, the challenge is to distribute measurements proportionally to the local variability—highly variable regions require higher measurement density—in order to minimize the so-called mapping error. If the

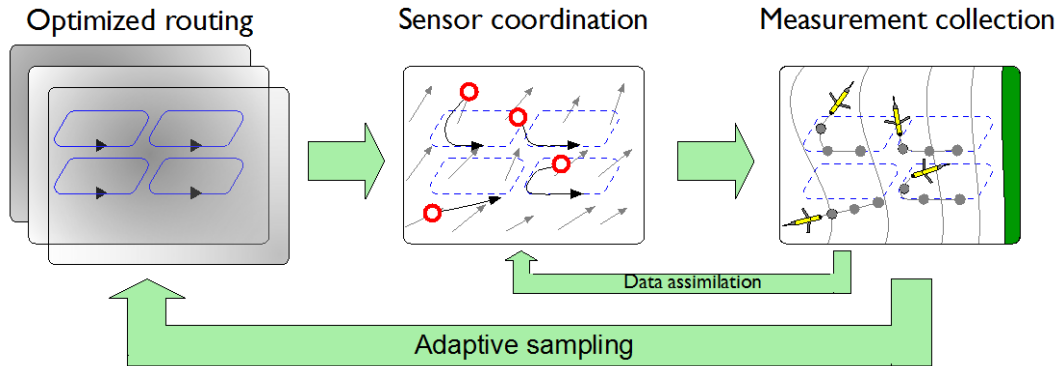


Figure 1. Dynamic, data-driven sampling uses (left) information-based metrics to optimize sensor routes, (middle) multi-vehicle control to stabilize the desired trajectories, and (right) nonlinear filtering to assimilate data; adaptive sampling refers to the re-optimization of sensor routes that occurs after data assimilation.

scales are unknown, the challenge is twofold: the scales themselves must be estimated while concurrently using the estimated scales for mapping. A dynamical characterization of the estimated process replaces the decorrelation statistics with the differential equations that govern the evolution in time of the process parameters. Dynamical descriptions permit the application of tools from systems theory including the concept of observability, which measures the sensitivity of the outputs of a system to perturbations of its states about a nominal value. The selection of where to collect sensor observations, i.e., where to route the sensor platforms, is thus formulated as the problem of maximizing observability of a nonlinear dynamical system. Since observability is traditionally a forward-looking metric, it is augmented with estimation uncertainty in order to account for locations of prior observations. Figure 1 depicts the adaptive-sampling feedback loop.

### Distributed Estimation of Spatiotemporal Fields

Observability-based optimization in path planning (Yu, 2011) and data assimilation (Krener, 2009) typically uses either a low-dimensional parameterized model or an empirical data-based representation of the unknown process; however, problems arise when neither a suitable model nor sufficient data are available. The novelty of the approach described here lies in the use of the observability of a low-dimensional model of an environmental vector field and a data-assimilation filter to guide the observability analysis via metrics from Bayesian experimental design. Observability- and information-based optimization of sampling trajectories yields a reliable and predictable capability for intelligent, mobile sensors. A dynamic, data-driven Bayesian nonlinear filter exploits noisy, low-fidelity and nonlinear measurements collected in a distributed manner by combining observations from individual or multiple sampling platforms.

Although methods exist for the optimization of sampling trajectories using distributed parameter estimation (Demetriou, 2010), optimal interpolation (Leonard, 2006), and heuristic approaches (Smith, 2010), an open question is how to rigorously characterize the variability of information content in an unknown spatiotemporal process and how to target observations in information-rich regions. The merit of the approach described here lies in the design of a statistical framework based on spatiotemporal estimation of nonstationary processes in meteorology (Karspeck, 2012) and geostatistics (Higdon, 1999). The framework extends the author's previous work in this area into multiple dimensions and builds a nonstationary statistical representation of a random process while simultaneously optimizing the sampling trajectories.

In oceanography, autonomous underwater vehicles are used as mobile sensors for adaptive sampling. Indeed, the concept of optimal experimental design was first applied to oceanographic sampling in the 1980's (Barth, 1990). The optimization of sensor placement and data collection has applications in fighting wildfires, finding perturbation sources in power

networks, and in spatial data collection for geostatistics. Perhaps it is not surprising, given to the range of these applications, that there are a variety of approaches advocated in the literature, including adaptation based on maximum *a posteriori* estimation; stochastic deployment policies; information-based methods; learning and artificial intelligence; deterministic methods with heuristic metrics; bio-inspired source localization and gradient climbing; and nonparametric Bayesian models. The results described here differ from prior work on adaptive sampling of dynamical systems and random processes in the novel application of nonlinear observability and control coupled with recursive Bayesian filtering to optimize sensor routing for environmental sampling.

One of our approaches to adaptive sampling in the ocean uses observability: a measure of how well the state variables of a control system can be determined by measurements of its outputs. Observability of a linear system (Hespanha, 2009) is characterized using the Kalman rank condition, which is a special case of the observability rank condition of a general, nonlinear system (Hermann, 1977). (A nonlinear system is called *observable* if two states are indistinguishable only if the states are identical.) Observability in data assimilation refers to the ability to determine the parameters of an unknown process from a time history of observations. Although standard observability tests give a binary answer (i.e., is the system observable or not), the degree of observability may be computed from the singular values of the observability gramian (Krener, 2009), which is a Hermitian matrix containing inner products of the system's outputs under systematic perturbations of the system's parameters about a nominal value. Computation of the empirical observability gramian requires only the ability to simulate the system, and is therefore particularly attractive for numerical optimization.

A second approach is based on classical estimation theory (Liebelt, 1967): optimal statistical interpolation of sensor observations to produce a stochastic estimate of an unknown random process, formerly known in meteorology and oceanography as objective analysis (Bretherton, 1976). Optimal interpolation also yields a measure of the uncertainty or error in the estimate, which can be used as a measure of estimator performance or skill (Leonard, 2007). It is common to compute estimation error under the assumption of stationarity of the spatial and temporal variability of the unknown process, although these assumptions may not be borne out in applications of interest. A stochastic process whose variability changes when shifted in time or space is called *nonstationary*, and methods exist to parametrize nonstationary processes in oceanography and geostatistics. Indeed, nonstationary-based strategies have been previously applied to mobile sensor networks, though not based on a principled control design.

### **Data-driven Adaptive Sampling: Measures of Observability**

The observability of a nonlinear system may be difficult to determine analytically, because it requires tools from differential geometry (Hermann, 1977). If the dynamical model of interest is solved numerically, it is justified to pursue numerical techniques for calculating the empirical observability gramian (Krener, 2009). The empirical observability gramian does not require linearization, which may fail to adequately model the input/output relationship of the nonlinear system over a wide range of operating conditions, but merely the ability to simulate the system. Indeed, the empirical observability gramian maps the input-output behavior of a nonlinear system more accurately than the observability gramian produced by linearization of the nonlinear system. The empirical observability gramian is a square matrix whose dimension matches the size of the state vector and whose  $(i,j)$ th entry represent the sensitivity of the output to infinitesimal perturbations about their nominal value of the corresponding  $i$  and  $j$  states or

unknown parameters. The observability of a nonlinear system is measured by calculating the unobservability index  $\nu$  of the empirical observability gramian. This index is used to score candidate trajectories for their anticipated information gain. Figure 2 depicts observability-based feedback loop, including a recursive Bayesian filter to assimilate noisy measurements from the observing platforms.

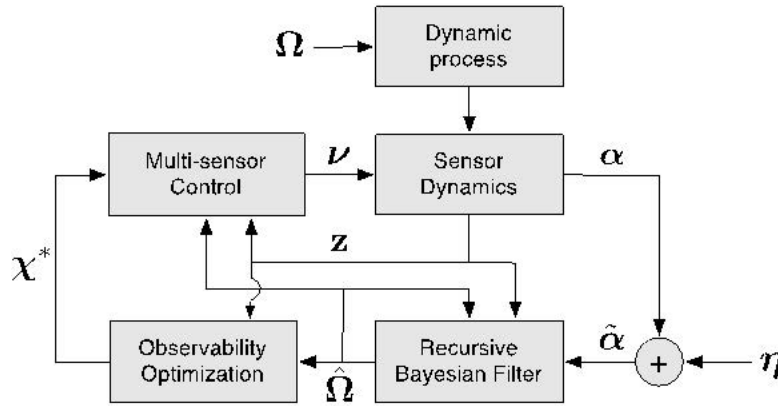


Figure 2. A schematic diagram of an observability-based sampling algorithm. A recursive Bayesian filter provides parameter estimates  $\hat{\Omega}$  from noisy measurements  $\tilde{\alpha}$ . The estimated parameters are used to calculate observability optimizing control parameters  $\chi^*$  that characterize the multi-sensor sampling formation.

### Data-driven Adaptive Sampling: Mapping Error

A spatiotemporal field is statistically described by its mean and the covariance function between any two points  $i$  and  $j$ . A covariance function is a positive-definite function that describes the variability of the field between the  $i$ th and  $j$ th location (Bennet, 2005). A field is stationary if its covariance function depends only on the relative position of  $i$  and  $j$  and it is nonstationary if it depends on  $i$  and  $j$  independently. There are a number of choices for the form of a nonstationary covariance function, e.g., Matern, rational quadratic, Ornstein-Uhlenbeck and squared-exponential forms (see Higdon, 1999). For a statistics-based sampling strategy, we require a covariance function that is a product of spatial- and temporal-covariance functions, such as a nonstationary squared exponential covariance function. In this case, the square roots of the diagonal elements are the spatial and temporal decorrelation scales of the field. (The

decorrelation scales are the spatial and temporal separations at which the covariance function evaluates to  $1/e$ .) For a stationary field, the decorrelation scales are constant, but for a nonstationary field they may vary in space and time. The covariance function is used to derive a coordinate transformation that clusters measurements in space-time regions with short decorrelation scales, and spreads measurements elsewhere, where the measurement demand is lower.

Statistics-based sensor routing seeks to provide optimal coverage of an estimated spatiotemporal field. The coverage is deemed optimal when the measurement density in space and time is proportional to the variability of the field. To determine when measurements are redundant, consider the footprint of a measurement, defined as the volume in space and time contained in an ellipsoid centered at the measurement location with principle axes equal to the decorrelation scales of the field. The goal is to design the vehicle trajectories so that the swaths created by the set of all measurement footprints cover the entire field with minimal overlaps or gaps, even when the decorrelation scales of the field vary. To determine the mapping error, we employ optimal interpolation (Bennett, 2005). The mapping error is the diagonal of the error covariance matrix. The average (resp. maximum) mapping error is computed by averaging (resp. finding the maximum of) all of the elements of the mapping error. Since the vehicles sample uniformly in time, the mapping error is minimized in a stationary field by traveling at maximum speed to place as many measurements as possible in the domain (Sydney, 2014). Figure 3 depicts the mapping error for a stationary field with correlation scale estimated by a Bayesian filter.

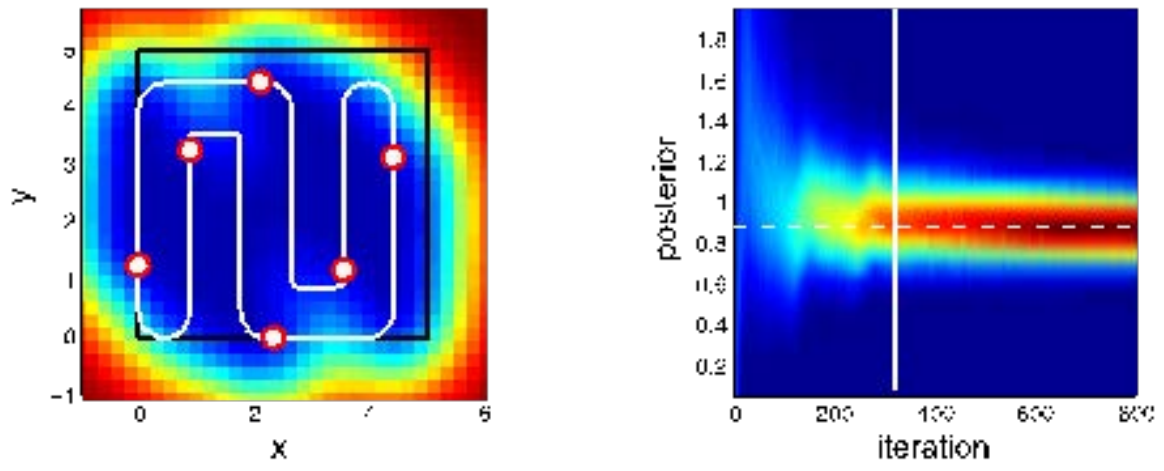


Figure 3. (left) Mapping error of a 2D stationary spatiotemporal field; (right) dynamic estimation of the decorrelation scale.



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